MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
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WRITTEN TESTS ON HIGHER MATHEMATICS and methodical instructions for their performance (4th module)

for students majoring in
141 Power Engineering, Electrical Engineering
and Electrical Mechanics

2022

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INTRODUCTION

The offered written tests on higher mathematics and methodical instructions for their performance correspond to the course «Higher Mathematics» taught to students majoring in Power Engineering, Electrical Engineering and Electrical Mechanics full-time in the fourth module of the two-semester course. This set of tasks can also be used for students in other technical majors who study higher mathematics in a two-semester course.

The manual contains tasks for two written tests and methodical instructions for their performance (fully solved standard options with all the necessary explanations). Each test is composed of ten options.

Written Test №1 covers the following topics of the fourth module of the course, namely: elements of probability theory, discrete random variables, continuous random variables. Written Test №2 covers the following topics of the fourth module of the course, namely: elements of mathematical statistics, sample processing, hypothesis testing, construction of confidence intervals.

At the end there is a list of recommended literature for preparation. It includes textbooks, as well as manuals on the performance of calculation tasks executed by students during the semester as homework. In addition to the methodological instructions for solving problems and examples of solutions, manual [14] also contains the necessary theoretical information and reference material (tables, literature, etc.).

Students perform each written test at the end of the study of the relevant module of the course. Written test is performed during classroom classes, it is designed for two academic hours.

The student chooses the option (the number of the variant) according to his number in the register list. The work is done in English in writing, preferably in a notebook in a cell. Note fields must be left blank. The name of the subject, major, group and course, surname, first name and patronymic of the student, name of the teacher who accepts the work should be indicated on the title page of the work.

While performing the work, the student should solve the offered tasks by the methods specified in the tasks, as well as make all the

necessary drawings (graphical solutions). The student must show the acquired theoretical knowledge of the course.

When evaluating a work, the indicator of its quality is, first of all, how the student independently and correctly solved the tasks and understood the content of the obtained solutions. After checking the work by teacher the student receives a certain number of rating points.

After checking and evaluating the work, solutions are analyzed and discussed. Thus, these tests are an important part of students' independent classroom work in practical classes; they help students assess their current level of knowledge. In addition, for the teacher these works is a method of diagnosis and control of learning material by students.

The materials of this manual can also be used for practical illustration of theoretical positions during the presentation of lecture material, during practical classes, as well as for modular control (its classroom part).

1 WRITTEN TEST №1

1.1 Methodical instructions for performance of Written Test №1 (solution of a standard option)

Option 0

Task 1. In a batch of 20 products, 5 products are defective. 6 products are chosen at random from the batch. Determine the probability that 2 of these 6 products will be defective.

Solution. Number of ways to select 6 products from 20:

$$n = C_{20}^{6} = \frac{20!}{6!14!} = 38760 .$$

This is the number of all possible outcomes.

Number of ways to select 2 defective products from 5 defective products:

$$C_5^2 = \frac{5!}{2!3!} = 10 .$$

Number of ways to select 6-2=4 non-defective products from 20-5=15 non-defective products:

$$C_{15}^4 = \frac{15!}{4!11!} = 1365 .$$

Then

$$m = C_5^2 \cdot C_{15}^4 = 13650 .$$

This is the number of outcomes favorable to the required event.

According to the classical definition of probability we have:

$$P = \frac{m}{n} = \frac{13650}{38760} = \frac{455}{1292} \approx 0.3522 .$$

Answer: $\frac{455}{1292} \approx 0.3522$.

Task 2. The probability that there will be no failures in the supply of raw materials during the working day is equal to 0.86. Find the probability that during a working week (five days):

a) failures will occur within three days;

b) failures will occur in no more than one day;

c) failures will occur on at least one day.

Solution. The conditions of the problem correspond to Bernoulli's scheme (sequence of independent trials). We have $n=5$, $q=0.86$ (probability of no failures will occur during the day), $p=1-q=0.14$ (probability of failures will occur during the day). Then the probability of k days will be with failures is calculated using the formula:

$$P_n(k)=C_n^k p^k q^{n-k}.$$

a) Then according to Bernoulli's formula we obtain (for $k=3$):

$$P_5(3)=C_5^3(0.14)^3(0.86)^{5-3}=\frac{5!}{2!3!}(0.14)^3\cdot(0.86)^2=$$
$$=0.020294624\approx 0.0203.$$

b) We have:

$$P(k\le 1)=P_5(0)+P_5(1)=$$
$$=C_5^0(0.14)^0(0.86)^{5-0}+C_5^1(0.14)^1(0.86)^{5-1}=$$
$$=\frac{5!}{5!0!}1\cdot(0.86)^5+\frac{5!}{4!1!}0{,}14\cdot(0.86)^4=$$
$$=0.8533327296\approx 0{,}8533.$$

c) Let's find the probability of the complementary event (the failures will not occur on any day):

$$P(k<1)=P_5(0)=C_5^0(0.14)^0(0.86)^{5-0}=\frac{5!}{5!0!}1\cdot(0.86)^5=$$
$$=0.4704270176\approx 0.4704.$$

Then the required probability is:

$$P(k\ge 1)=1-P(k<1)=1-0.4704270176=$$
$$=0.5295729824\approx 0.5296.$$

Answer: а) 0.0203;
б) 0.8533;
в) 0.5296.

Task 3. 110 students study on the course. Of them, 10 students study excellently, 50 students study well, 15 students study unsatisfactory, and the rest study mediocre. The probability of solving the problem for an excellent student is 0.96, for a student who studies well – 0.75, for a student who studies mediocre – 0.46, for a student who studies unsatisfactory – 0.15.

a) What is the probability of solving the problem for a randomly selected student of the course?

b) A randomly selected student did not solve the problem. What is the probability that this is a student who studies well?

Solution. The conditions of the problem correspond to the situation when total probability and Bayes' formula are used (a posteriori reestimations of hypotheses probabilities). Let B_i, $i = 5,4,3,2$, be a hypothesis that a randomly selected student studies excellently (well, mediocre, unsatisfactory respectively). According to the condition of the problem we have:

$$P(B_5) = \frac{10}{110} = \frac{1}{11}, \qquad P(B_4) = \frac{50}{110} = \frac{5}{11},$$

$$P(B_3) = \frac{110-10-50-15}{110} = \frac{35}{110} = \frac{7}{22}, \qquad P(B_2) = \frac{15}{110} = \frac{3}{22}.$$

Let A be an event that means that a randomly selected student solved the problem. The probability of this event given the hypothesis B_i, $i = 5,4,3,2$, is fulfilled, denote $P_{B_i}(A)$ (conditional probability). According to the condition of the problem we have

$$P_{B_5}(A) = 0.96, \qquad P_{B_4}(A) = 0.75,$$

$$P_{B_3}(A) = 0.46, \qquad P_{B_2}(A) = 0.15.$$

a) The probability of solving the problem for a randomly selected student is (formula of a total probability):

$$P(A) =$$

$$= P_{B_2}(A) \cdot P(B_2) + P_{B_3}(A) \cdot P(B_3) + P_{B_4}(A) \cdot P(B_4) + P_{B_5}(A) \cdot P(B_5) =$$

$$= 0.15 \cdot \frac{3}{22} + 0.46 \cdot \frac{7}{22} + 0.75 \cdot \frac{5}{11} + 0.96 \cdot \frac{1}{11} = \frac{1309}{2200} = 0.595.$$

b) A randomly selected student did not solve the problem. The probability that this is a student who studies well (probability of realization of the hypothesis B_4, given event $\overline{A}$ has already occurred, it's Bayes' formula):

$$P_{\overline{A}}(B_4) = \frac{P(B_4) \cdot \mathrm{P}_{B_4}(\overline{A})}{\mathrm{P}(\overline{A})} = \frac{(1 - 0{,}75) \cdot \frac{5}{11}}{\left(1 - \frac{1309}{2200}\right)} = \frac{250}{891} \approx 0.2806.$$

Answer: a) $\frac{1309}{2200} = 0.595$;

b) $\frac{250}{891} \approx 0.2806$

Task 4. The probability distribution of a discrete random variable X has the next form:

x_i	-1	1	3	5	7	8	9	10	11	16
p_i	0.1	0.2	0.1	p_4	0.1	0.15	0.1	0.05	0.1	0.05

Find probability p_4, expected value EX, variance VX and standard deviation $\sigma(X)$. Sketch the distribution function $F(x)$.

Solution. We can find the probability p_4 from the normalization condition $\sum p_i = 1$:

$$p_4 = 1 - (0.1 + 0.2 + 0.1 + 0.1 + 0.15 + 0.1 + 0.05 + 0.1 + 0.05) = 0.05.$$

Expected value:

$$EX = \sum x_i\, p_i =$$
$$= (-1)\cdot 0.1 + 1\cdot 0.2 + 3\cdot 0.1 + 5\cdot 0.05 + 7\cdot 0.1 +$$
$$+ 8\cdot 0.15 + 9\cdot 0.1 + 10\cdot 0.05 + 11\cdot 0.1 + 16\cdot 0.05 = 5.85.$$

Variance:

$$VX = \sum x_i^2\, p_i - (EX)^2 =$$
$$= (-1)^2\cdot 0.1 + 1^2\cdot 0.2 + 3^2\cdot 0.1 + 5^2\cdot 0.05 + 7^2\cdot 0.1 +$$
$$+ 8^2\cdot 0.15 + 9^2\cdot 0.1 + 10^2\cdot 0.05 + 11^2\cdot 0.1 + 16^2\cdot 0.05 - 5.85^2 =$$
$$= 54.95 - 34.2225 = 20.7275.$$

Standard deviation:

$$\sigma(X) = \sqrt{VX} = \sqrt{20.7275} \approx 4.5527.$$

Let's find the distribution function (by summing up the accumulated probabilities):

$$F(x) = \begin{cases} 0; & x \le -1 \\ 0.1; & -1 < x \le 1 \\ 0.3; & 1 < x \le 3 \\ 0.4; & 3 < x \le 5 \\ 0.45; & 5 < x \le 7 \\ 0.55; & 7 < x \le 8 \\ 0.7; & 8 < x \le 9 \\ 0.8; & 9 < x \le 10 \\ 0.85; & 10 < x \le 11 \\ 0.95; & 11 < x \le 16 \\ 1; & x > 16 \end{cases}$$

The graph of the distribution function $F(x)$ is shown in Fig. 2.1.

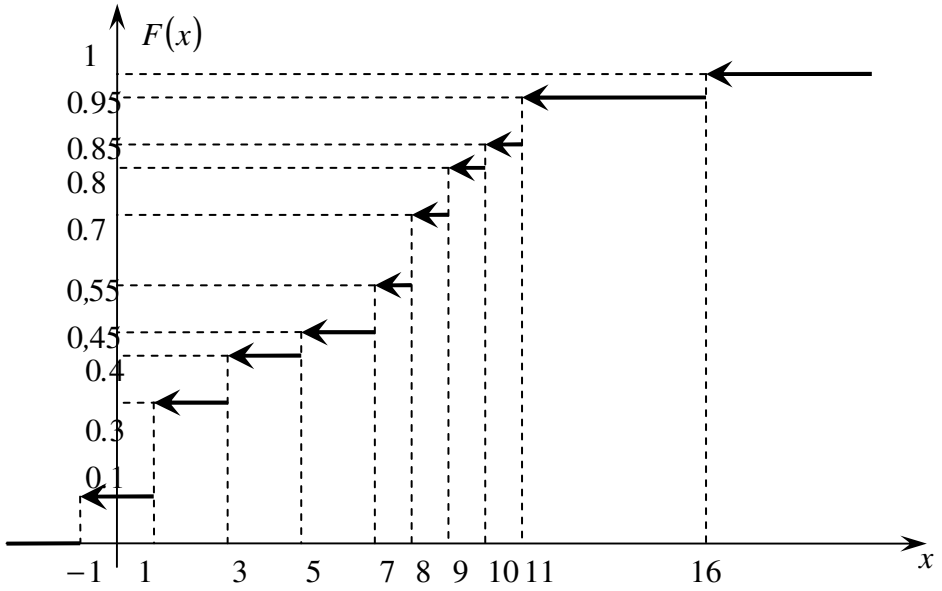

Fig. 2.1

Answer: $p_4 = 0.05$;
$EX = 5.85$;
$VX = 20.7275$;
$\sigma(X) \approx 4.5527$;
$F(x)$ – Fig. 2.1.

Task 5. The random measurement error X is normally distributed with a standard deviation of $\sigma = 5$. The systematic error (expected value) is $a = 4$. Find the probability of the measurement error X will take a value from the interval $(\alpha; \beta)$, $\alpha = 2, \beta = 11$.

Solution. The required probability can be found by the formula:

$$P(\alpha < X < \beta) = \Phi\left(\frac{\beta - a}{\sigma}\right) - \Phi\left(\frac{\alpha - a}{\sigma}\right),$$

where $\Phi(x)$ is the Laplace function, the value of which can be found in the tables (see Appendix A). We have:

$$P(2 < X < 11) = \Phi\left(\frac{11-4}{5}\right) - \Phi\left(\frac{2-4}{5}\right) = \Phi(1.4) - \Phi(-0.4) =$$

$$= \Phi(1.4) + \Phi(0.4) = 0.4192 + 0.1554 = 0.5746.$$

We used the oddity of the function $\Phi(x)$.

Answer: 0.5746.

1.2 Options of Written Test №1

Substitute your registration number instead of N.

Task 1. The director of the company concluded $N+7$ contracts. Five of them, contrary to the advice of a lawyer, he concluded in violation of tax laws. Find the probability that during tax verification, among the four contracts randomly selected, two will be free of violation of the law.

Task 2. A cadet fires $N+3$ independent shots at a target with a hit probability of 0.2. Find the probability of:

a) two target hits;
b) at least two target hits;
c) no more than one target hit.

Task 3. The probability that the bank customer will not repay the loan during the period of economic growth is $\frac{1}{N+20}$, and during the period of the economic crisis this probability is $\frac{1}{N}$. The economic forecast claims that the probability of economic growth is 0.45. What is the probability that a randomly selected bank customer will not return the loan?

What is the probability that the economy is in a state of crisis given a bank customer has not returned a loan?

Task 4. The probability distribution of a discrete random variable X has the next form:

x_i	-2	-1	0	1	N
p_i	0.2	0.1	0.2	p_4	0.1

Find probability p_4, expected value EX, variance VX and standard deviation $\sigma(X)$. Sketch the distribution function $F(x)$.

Task 5. The packaging machine packs laundry detergent in bags, the average weight of which is $930g$, and the standard deviation $(20+0{,}1\cdot N)g$. What proportion (percentage) of packages will weight less than $900g$?

2 WRITTEN TEST №2

2.1 Methodical instructions for performance of Written Test №2 (solution of a standard option)

Option 0

Task 1. For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	1	2	3	4	6	7	8
n_i	1	2	2	5	2	2	1

Solution. We have a sample of $n = 15$ objects. Let's calculate the numerical characteristics of the sample.

The sample mean:

$$\overline{x} = \frac{1}{n}\sum x_i n_i = \frac{1}{15}\left(1\cdot 1 + 2\cdot 2 + ... + 8\cdot 1\right) \approx 4.3333 .$$

The sample variance:

$$V = \frac{1}{n}\sum x_i^2 n_i - \left(\overline{x}\right)^2 =$$

$$= \frac{1}{15}\left(1^2\cdot 1 + 2^2\cdot 2 + ... + 8^2\cdot 1\right) - 4.3333^2 = 3.9556 .$$

The standard deviation:

$$\sigma_{sample} = \sqrt{V} = \sqrt{3.9556} = 1.9889 .$$

The unbiased variance:

$$s^2 = \frac{n}{n-1}V = \frac{15}{14}\cdot 3.9556 = 4.2381 .$$

The unbiased sample standard deviation:

$$s = \sqrt{\frac{n}{n-1}V} = \sqrt{\frac{15}{14} \cdot 3.9556} = 2.0587 .$$

Let's find the sample mode. This is the variant that corresponds to the maximum frequency (see Table):

$$Mo = 4 .$$

Let's find the sample median. This is the middle of the variation series:

$$Me = 4 .$$

The coefficient of variation:

$$\nu = \frac{\sqrt{V}}{\bar{x}} \cdot 100\% = \frac{\sqrt{3.9556}}{4.3333} \cdot 100\% = 45.90\% .$$

Answer: $\overline{x} = 4.3333$;

$V = 3.9556$; $\sigma_{sample} = 1.9889$;

$s^2 = 4.2381$; $s = 2.0587$;

$Mo = 4$; $Me = 4$; $\nu = 45.90\%$.

Task 2. Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

Solution. Since the population variance is unknown, the confidence interval for the expected value a is sought by the formula

$$\bar{x} - \delta < a < \bar{x} + \delta ,$$

where

$$\delta = t_\gamma \frac{s}{\sqrt{n}} .$$

We look for the critical point of the Student distribution $t_\gamma = t(n, \gamma)$ according to the table (see Appendix B): $t_\gamma = t(n = 15, \gamma = 0{,}95) = 2.14$. We have:

$$\delta = t_\gamma \frac{s}{\sqrt{n}} = 2.14 \cdot \frac{2.0587}{\sqrt{15}} = 1.1375 .$$

Then we get the confidence interval:

$$4.3333 - 1.1375 < a < 4.3333 + 1.1375 ,$$
$$3.1958 < a < 5.4708 .$$

Answer: $3.1958 < a < 5.4708$.

Task 3. A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	100	105	110	115	120	125	130
n_i	1	2	5	28	26	6	2
n_i'	1	2	10	24	24	9	1

Solution. Let's compare empirical and theoretical frequencies. We compose a calculation table from which we find the chi-squared test statistic:

$$\chi^2 = \sum_i \frac{(n_i - n_i')^2}{n_i'} .$$

Here the frequencies of the first two intervals were united, since they are too small.

The number of class intervals is $s = 6$ (since two intervals were united). The number of parameters of the normal distribution determined from the sample is $r = 2$. Then the degrees of freedom are $k = s - r - 1 = 6 - 2 - 1 = 3$.

i	n_i	n_i'	$n_i - n_i'$	$(n_i - n_i')^2$	$\frac{(n_i - n_i')^2}{n_i'}$
1	1	1			
2	2 3	2 3	0	0	0.0000
3	5	10	-5	25	2.5000
4	28	24	4	16	0.6667
5	26	24	2	4	0.1667
6	6	9	-3	9	1.0000
7	2	1	1	1	1.0000
$\sum$	70				$\chi^2 = 5.3334$

For the given significance level $\alpha = 0.01$ and the degrees of freedom $k = 3$, we find the threshold χ^2 value. This value can be found in table (see Appendix C):

$$\chi^2_{thr} = \chi^2(0.01;\ 3) = 11.3.$$

Since $\chi^2 < \chi^2_{thr}$, we do not reject the null hypothesis of the normal distribution of the general population. The discrepancy between empirical and theoretical frequencies is random. The null hypothesis is sustained with the significance level $\alpha = 0.01$, the observed data do not contradict the hypothesis of a normal distribution of the general population.

Answer: the hypothesis is not rejected.

2.2 Options of Written Test №2

Option 1

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	18	20	22	24	26
n_i	5	6	10	4	5

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	2	3	4	5	6	7	8
n_i	7	15	20	25	18	13	5
n_i'	5	14	19	26	20	12	6

Option 2

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	25	30	35	40	45

n_i	8	18	42	20	12

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	5,5	10,5	15,5	20,5	25,5	30,5	35,5
n_i	6	8	15	40	16	8	7
n_i'	5	10	20	27	21	11	6

Option 3

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	25	35	45	55	65
n_i	10	15	8	5	2

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	2	3	4	5	6	7	8
n_i	7	15	20	25	18	13	5
n_i'	5	14	19	26	20	12	6

Option 4

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	25	30	35	40	45
n_i	3	8	4	3	2

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	100	105	110	115	120	125
n_i	5	16	24	13	16	8
n_i'	6	11	18	20	17	10

Option 5

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	380	390	400	410	420
n_i	4	5	6	2	3

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	16	18	20	22	24	26	28
n_i	5	7	9	10	17	15	11
n_i'	7	9	12	14	12	11	9

Option 6

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	240	250	260	270	280
n_i	4	6	5	3	2

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	1	3	5	7	9	10	11	12
n_i	5	10	35	70	100	80	20	10
n_i'	6	13	37	78	95	65	27	9

Option 7

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	30	33	35	37	40
n_i	11	15	28	14	12

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	2	4	6	8	10	11	12
n_i	6	16	20	25	18	14	4
n_i'	5	14	19	26	20	12	6

Option 8

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	30	40	50	60	70
n_i	12	13	25	11	9

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	5	6	7	8	9	10	11
n_i	7	15	21	24	19	12	5
n_i'	5	14	19	26	20	12	6

Option 9

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	25	30	33	35	40
n_i	2	3	8	4	3

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	5,5	10,5	15,5	20,5	25,5	30,5	35,5
n_i	6	9	14	39	17	8	7
n_i'	5	10	20	27	21	11	6

Option 10

1) For a given sample, calculate the sample mean, mode, median, variance, standard deviation, unbiased variance, unbiased standard deviation, coefficient of variation.

x_i	8	9	10	11	12
n_i	7	10	15	12	6

2) Assuming that in the previous problem the sample was taken from a normal distribution with unknown parameters, find a 95% confidence interval for the expected value of the random variable *X*.

3) A sample is obtained from the general population. Theoretical frequencies n_i' were calculated under the assumption of a normal

distribution of the trait. Using Pearson's χ^2 test with the significance level of 0.01, sustain or reject the hypothesis of the normal distribution of the general population.

x_i	100	105	110	115	120	125
n_i	5	15	23	15	16	8
n_i'	6	11	18	20	17	10

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Appendix A

Values of the Laplace function $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0,00	0,0000	0,19	0,0753	0,38	0,1480	0,57	0,2157
0,01	0,0040	0,20	0,0793	0,39	0,1517	0,58	0,2190
0,02	0,0080	0,21	0,0832	0,40	0,1554	0,59	0,2224
0,03	0,0120	0,22	0,0871	0,41	0,1591	0,60	0,2257
0,04	0,0160	0,23	0,0910	0,42	0,1628	0,61	0,2291
0,05	0,0199	0,24	0,0948	0,43	0,1664	0,62	0,2324
0,06	0,0239	0,25	0,0987	0,44	0,1700	0,63	0,2357
0,07	0,0279	0,26	0,1026	0,45	0,1736	0,64	0,2389
0,08	0,0319	0,27	0,1064	0,46	0,1772	0,65	0,2422
0,09	0,0359	0,28	0,1103	0,47	0,1808	0,66	0,2454
0,10	0,0398	0,29	0,1141	0,48	0,1844	0,67	0,2486
0,11	0,0438	0,30	0,1179	0,49	0,1879	0,68	0,2517
0,12	0,0478	0,31	0,1217	0,50	0,1915	0,69	0,2549
0,13	0,0517	0,32	0,1255	0,51	0,1950	0,70	0,2580
0,14	0,0557	0,33	0,1293	0,52	0,1985	0,71	0,2611
0,15	0,0596	0,34	0,1331	0,53	0,2019	0,72	0,2642
0,16	0,0636	0,35	0,1368	0,54	0,2054	0,73	0,2673
0,17	0,0675	0,36	0,1406	0,55	0,2088	0,74	0,2703
0,18	0,0714	0,37	0,1443	0,56	0,2123	0,75	0,2734

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0,76	0,2764	0,99	0,3389	1,22	0,3883	1,45	0,4265
0,77	0,2794	1,00	0,3413	1,23	0,3907	1,46	0,4279
0,78	0,2823	1,01	0,3438	1,24	0,3925	1,47	0,4292
0,79	0,2852	1,02	0,3461	1,25	0,3944	1,48	0,4306
0,80	0,2881	1,03	0,3485	1,26	0.3952	1,49	0,4319
0,81	0,2910	1,04	0,3508	1,27	0.3980	1,50	0,4332
0,82	0.2939	1,05	0,3531	1,28	0,3997	1,51	0,4345
0,83	0,2967	1,06	0,3554	1,29	0,4015	1,52	0,4357
0,84	0,2995	1,07	0,3577	1,30	0,4032	1,53	0,4370
0,85	0,3023	1,08	0,3599	1,31	0,4049	1,54	0,4382
0,86	0,3051	1,09	0,3621	1,32	0,4066	1,55	0,4394
0,87	0,3078	1,10	0,3643	1,33	0,4082	1,56	0,4406
0,88	0,3106	1,11	0,3665	1,34	0,4099	1,57	0,4418
0,89	0,3133	1,12	0,3686	1,35	0,4115	1,58	0,4429
0,90	0,3159	1,13	0,3708	1,36	0,4131	1,59	0,4441
0,91	0,3186	1,14	0,3729	1,37	0,4147	1,60	0,4452
0,92	0,3212	1,15	0,3749	1,38	0,4162	1,61	0,4463
0,93	0,3238	1,16	0,3770	1,39	0,4177	1,62	0,4474
0,94	0,3264	1,17	0,3790	1,40	0,4192	1,63	0,4484
0,95	0,3289	1,18	0,3810	1,41	0,4207	1,64	0,4495
0,96	0,3315	1,19	0,3830	1,42	0,4222	1,65	0,4505
0,97	0,3340	1,20	0,3849	1,43	0,4230	1,66	0,4515
0,98	0,3365	1,21	0,3869	1,44	0,4251	1,67	0,4525

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
1,68	0,4535	1,91	0,4719	2,28	0,4887	2,74	0,4969
1,69	0,4545	1,92	0,4726	2,30	0,4893	2,76	0,4971
1,70	0,4554	1,93	0,4732	2,32	0,4898	2,78	0,4973
1,71	0,4564	1,94	0,4738	2,34	0,4904	2,80	0,4974
1,72	0,4573	1,95	0,4744	2,36	0,4909	2,82	0,4976
1,73	0,4582	1,96	0,475	2,38	0,4913	2,84	0,4977
1,74	0,4591	1,97	0,4756	2,40	0,4918	2,86	0,4979
1,75	0,4599	1,98	0,4761	2,42	0,4922	2,88	0,4980
1,76	0,4608	1,99	0,4767	2,44	0,4927	2,90	0,4981
1,77	0,4616	2,00	0,4772	2,46	0,4931	2,92	0,4982
1,78	0,4525	2,02	0,4783	2,48	0,4934	2,94	0,4984
1,79	0,4633	2,04	0,4793	2,50	0,4938	2,96	0,4985
1,80	0,4641	2,06	0,4803	2,52	0,4941	2,98	0,4985
1,81	0,4649	2,08	0,4812	2,54	0,4945	3,00	0,49865
1,82	0,4656	2,10	0,4821	2,56	0,4948	3,20	0,49931
1,83	0,4664	2,12	0,4830	2,58	0,4951	3,40	0,49966
1,84	0,4671	2,14	0,4838	2,60	0,4953	3,60	0,499841
1,85	0,4678	2,16	0,4846	2,62	0,4956	3,80	0,499928
1,86	0,4686	2,18	0,4854	2,64	0,4959	4,00	0,499968
1,87	0,4693	2,20	0,4861	2,66	0,4961	4,50	0,499997
1,88	0,4699	2,22	0,4868	2,68	0,4963	5,00	0,499997
1,89	0.4706	2,24	0,4875	2,70	0,4965		
1,90	0,4713	2,26	0,4881	2,72	0,4967		

Appendix B

Values of $t_\gamma = t(\gamma; n)$

n, sample size	γ, confidence level				
	0,90	**0,95**	**0,98**	**0,99**	**0,999**
6	2,02	2,57	3,36	4,03	6,87
7	1,94	2,45	3,14	3,71	5,96
8	1,89	2,36	3,00	3,50	5,41
9	1,86	2,31	2,90	3,36	5,04
10	1,83	2,26	2,82	3,25	4,78
11	1,81	2,23	2,76	3,17	4,59
12	1,80	2,20	2,72	3,11	4,44
13	1,78	2,18	2,68	3,05	4,32
14	1,77	2,16	2,65	3,01	4,22
15	1,76	2,14	2,62	2,98	4,14
16	1,75	2,13	2,60	2,95	4,07
17	1,75	2,12	2,58	2,92	4,02
18	1,74	2,11	2,57	2,90	3,97
19	1,73	2,10	2,55	2,88	3,92
20	1,73	2,09	2,54	2,86	3,88
21	1,72	2,09	2,53	2,85	3,85
22	1,72	2,08	2,52	2,83	3,82
23	1,72	2,07	2,51	2,82	3,79
24	1,71	2,07	2,50	2,81	3,77
25	1,71	2,06	2,49	2,80	3,75
26	1,71	2,06	2,49	2,79	3,73
27	1,71	2,06	2,49	2,78	3,71
28	1,70	2,05	2,47	2,77	3,69
29	1,70	2,05	2,47	2,76	3,67
30	1,70	2,05	2,46	2,76	3,66
31	1,70	2,04	2,46	2,75	3,65
41	1,68	2,02	2,42	2,70	3,55
51	1,68	2,01	2,40	2,68	3,50
61	1,67	2,00	2,39	2,66	3,46
121	1,66	1,98	2,36	2,62	3,37
∞	1,64	1,96	2,33	2,58	3,29

Appendix C

Critical points of chi-square distribution

k, number of degrees of freedom	α, significance level								
	0,99	**0,95**	**0,90**	**0,10**	**0,05**	**0,025**	**0,01**	**0,005**	**0,001**
1	0,0002	0,004	0,02	2,71	3,84	5,02	6,63	7,88	10,8
2	0,002	0,10	0,21	4,61	5,99	7,38	9,21	10,6	13,8
3	0,12	0,35	0,58	6,25	7,81	9,35	11,3	12,8	16,3
4	0,30	0,71	1,06	7,78	9,49	11,1	13,3	14,9	18,5
5	0,55	1,15	1,61	9,24	11,1	12,8	15,1	16,7	20,5
6	0,87	1,64	2,20	10,6	12,6	14,4	16,8	18,5	22,5
7	1,24	2,17	2,83	12,0	14,1	16,0	18,5	20,3	24,3
8	1,65	2,73	3,49	13,4	15,5	17,5	20,1	22,0	26,1
9	2,09	3,33	4,17	14,7	16,9	19,0	21,7	23,6	27,9
10	2,56	3,94	4,87	16,0	18,3	20,5	23,2	25,2	29,6
11	3,05	4,57	5,58	17,3	19,7	21,9	24,7	26,8	31,3
12	3,57	5,23	6,30	18,5	21,0	23,3	26,2	28,3	32,9
13	4,11	5,89	1,04	19,8	22,4	24,7	27,7	29,8	34,5
14	4,66	6,57	7,79	21,1	23,7	26,1	29,1	31,3	36,1
15	5,23	7,26	8,55	22,3	25,0	27,5	30,6	32,8	37,7
16	5,81	7,96	9,31	23,5	26,3	28,8	32,0	34,3	39,3
17	6,41	8,67	10,1	24,8	27,6	30,2	33,4	35,7	40,8
18	7,01	9,39	10,9	26,0	28,9	31,5	34,8	37,2	42,3
19	7,63	10,1	11,7	27,2	30,1	32,9	36,2	38,6	43,8
20	8,26	10,9	12,4	28,4	31,4	34,2	37,6	40,0	45,3
21	8,90	11,6	13,2	29,6	32,7	35,5	38,9	41,4	46,8
22	9,54	12,3	14,0	30,8	33,9	36,8	40,3	42,8	48,3
23	10,2	13,1	14,8	32,0	35,2	38,1	41,6	44,2	49,7
24	10,9	13,8	15,7	33,2	36,4	39,4	43,0	45,6	51,2
25	11,5	14,6	16,5	34,4	37,7	40,6	44,3	46,9	52,6
26	12,2	15,4	17,3	35,6	38,9	41,9	45,6	48,3	54,1
27	12,9	16,2	18,1	36,7	40,1	43,2	47,0	49,6	55,5
28	13,6	16,9	18,9	37,9	41,3	44,5	48,3	51,0	56,9
29	14,3	17,7	19,8	39,1	42,6	45,7	49,6	52,3	58,3
30	15,0	18,5	20,6	40,3	43,8	47,0	50,9	53,7	59,7