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Propagation and diffraction of thermo-elastic waves in a piecewise homogeneous strip composed of isotropic materials with different mechanical properties are investigated. One of the surfaces of the structure is subjected to thermal and mechanical shock. It is assumed that the velocity of heat propagation is finite. The equations of coupled thermo-elasticity and conditions of the solutions conjugation for the layers contact are adduced. The problem is solved numerically using the method of characteristics. The calculation results for a homogeneous strip are in good agreement with those obtained by other methods. The phenomena of reflection and transmission of thermo-elastic waves at the interface between layers are analyzed.

Key words: *thermo-elastic waves, stresses, velocity, method of characteristics.*

Introduction

Currently, there is a growing number of published articles and books, which generalize the theory of elasticity and plasticity for non-isothermal deformations [1, 2]. In particular, this is due to important problems arising in the development of new composite structures, operating in conditions of high-speed transient thermo-mechanical loads, when it is necessary to take into account coupling between fields of strain and temperature fields. Thermo-elastic stresses in composite materials can cause cracking, delamination, decrease of stiffness, and thin-walled components of multilayer structures result in thermal bulging.

Objective

To assess the strength and the bearing capacity of geometrically and physically non-homogeneous multilayer structures it is necessary to establish methods of calculation, allowing to carry out numerical experiments to identify areas most disposed to damage.

Formulation of the Problem

The object of the study is two linearly elastic anisotropic rigidly fastened together layers of thicknesses h_1 and h_2 with different mechanical properties (fig. 1). Normal intensity compressive forces p_0 are applied to the surface $x=0$ and simultaneously heat flow $q=-\overline{q}$ is supplied, where $\overline{q}$ is the heat flux density in the direction of outer normal to the surface $x=0$ of domain $x\geq 0$. It is assumed that the surfaces of the two-layer strip $x=0$ and $x=x_2$ are thermally insulated, and that the final speed of heat propagation is finite. For this task, the system of one-dimensional coupled increments of thermo-elasticity, valid for the same order of thermal and elastic deformations in the dimensionless notation has the form:

$$\frac{\partial^2\sigma}{\partial\xi^2}-\frac{1}{c_1^2}\frac{\partial^2\sigma}{\partial\tau^2}-\frac{\partial^2\Theta}{\partial\tau^2}=0, \qquad (1)$$

$$\frac{\partial^2\Theta}{\partial\xi^2}-\frac{1}{c_2^2}\frac{\partial^2\Theta}{\partial\tau^2}=(1+\varepsilon)\frac{\partial\Theta}{\partial\tau}+\varepsilon\frac{\partial\sigma}{\partial\tau}, \qquad (2)$$

Coupling of dimensionless and dimensional quantities is carried out according to formulas:

$\tau=c^2t/l$; $\xi=c\,t/l$; $c^2=(\lambda+2\mu)/\rho$; $\Theta=a(T-T_0)$; $\sigma=\sigma_x(1-2\nu)/E$; $c_1^2=1$; $c_2^2=c^2/V_0^2$; $V_0^2=a/\tau_0$; $a=\lambda_0/(c\rho)$.

Coefficient of connection is

$$\varepsilon=\frac{(3\lambda+2\mu)^2a_t^2T_0}{(\lambda+2\mu)c_\varepsilon}=\frac{(1+\nu)}{(1-\nu)}\frac{a_t^2ET_0}{(1-2\nu)c_\varepsilon}.$$

Where t is time; σ_x is stress; T, T_0 are current and initial temperatures correspondingly; τ_0 is relaxation time of heat flux [3, 4]; λ, μ are Lame coefficients; ν is Poisson coefficient; E is modulus of elasticity; ρ is density; c is a velocity of longitudinal wave propagation; α is coefficient of thermal

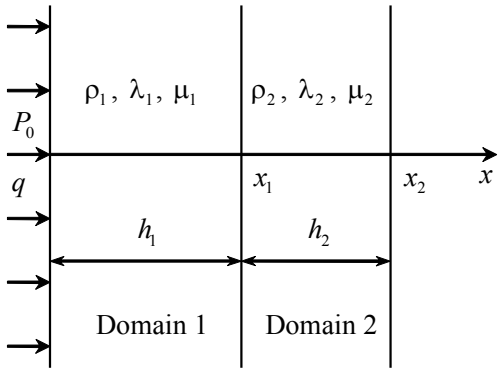

Fig. 1. Two-layer strip effected by thermo-mechanical shock

conductivity; $l = h_1 + h_2$ is thickness of layer.

The unknown quantities and parameters relevant to mechanical properties of a certain domain 1 or 2 (fig. 1) in what follows will be marked by the inferior index i ($i = 1, 2$) if necessary.

This problem is reduced to solving the system of equations (1), (2), and coupling σ and θ, with the initial

$$\sigma = \frac{\partial\sigma}{\partial t} = \theta = \frac{\partial\theta}{\partial\tau} = 0 \quad \text{at} \quad \tau = 0, \qquad (3)$$

boundary mechanical

$$\sigma = -p_0 f(\tau) \ \text{ at } \ \xi = 0; \ \sigma = 0 \ \text{ at } \ \xi = 1 \qquad (4)$$

and boundary thermal conditions

$$\frac{\partial\theta}{\partial\xi} = q\varphi(\tau) \ \text{ at } \ \xi = 0; \ \frac{\partial\theta}{\partial\xi} = 0 \ \text{ at } \ \xi = 1.$$

Here $f(\tau)$ and $\varphi(\tau)$ are specified laws of loading change. In addition, the consistency conditions at the boundary interface of two domains 1 and 2 are taken into account for $\xi = h_1 / l$:

$$\left.\frac{\partial\sigma}{\partial\tau}\right|_1 = \left.\frac{\partial\sigma}{\partial\tau}\right|_2; \quad \left.\frac{\partial\theta}{\partial\tau}\right|_1 = \left.\frac{\partial\theta}{\partial\tau}\right|_2. \qquad (6)$$

Solution Methods

The system of hyperbolic equations (1), (2) is solved numerically using the method of characteristics [5–7]. The equations of the characteristics and the relations on them have the form:

along $\frac{\partial\xi}{\partial\tau} = \pm c_1 = \pm 1$ the relation is performed

$$d\left(\frac{\partial\sigma}{\partial\tau}\right) \mp d\left(\frac{\partial\sigma}{\partial\xi}\right) = 0;$$

along $\frac{\partial\xi}{\partial\tau} = \pm c_2$ the relation is performed

$$d\left(\frac{\partial\theta}{\partial\tau}\right) \mp c_2 \cdot d\left(\frac{\partial\theta}{\partial\xi}\right) \pm c_2 \cdot \left((1+\varepsilon)\frac{\partial\theta}{\partial\tau} + \varepsilon\frac{\partial\sigma}{\partial\tau}\right) d\xi = 0.$$

For the calculations in domains 1 and 2 the grid formed by the family of characteristics $d\xi / d\tau = \pm 1$ (it is assumed $c = \max\left\{c|_1, c|_2\right\}$) is constructed. In fact it is necessary, as the other characteristic lines have a steeper slope. For computing the unknowns in the internal nodes of the grid and on the boundary a standard procedure is used [5–6]. The solution at the contact points of the two domains is constructed as follows [7]. Formally, a point belonging to the line $x = h_1 / l$ is considered as consisting of two points, one of which belongs to domain 1, and the other – to the domain 2. When the point belongs to the domain 1, integration along characteristics passing outside the domain 1 is excluded. On the other hand, this point belongs to the domain 2, and it is dealt with similarly as in the previous case. Obtained equations are complemented with the conditions of the contact (6).

The results of calculations

To check the operation of a computer system the problem for a semi-infinite homogeneous strip ($0 \le \xi < \infty$) with the initial conditions (3) and the boundary conditions (4) is solved, (5) at $\varphi(\tau) = 0$

$$f(\tau) = H(\tau) - H(\tau - \tau_1) = \begin{cases} 1 & \text{при} \quad 0 \le \tau \le \tau_1, \\ 0 & \text{при} \quad \tau > \tau_1. \end{cases}$$

Cases of mechanical shock effect only and thermal shock effect only have also been considered. When calculating the time integration step is assumed to be equal to $\Delta\tau = 0{,}01$, value $\tau_1 = 0{,}03$. The results of calculations for the layer of steel and a layer of a polymer material (butyral resin), mechanical and thermal properties of which are given in [3] are in good agreement with the results of [3, 4]. For a two-layer structure of the considered materials the load is specified in the form $f(\tau) = \tau / \exp(\tau)$, and $\varphi(\tau) = 0$. The dimensionless width of the first layer is equal to 0.5, and the second – 0.125. Coefficients are $\varepsilon|_1 = 0{,}0114$, $\varepsilon|_2 = 0{,}482$.

For $\tau_0 = 10^{-9}[c]$ velocity is $c_2 = 0{,}03$. The stress distribution along atwo-layer strip thickness at the time instants $\tau = 0{,}5$, $\tau = 0{,}75$ and $\tau = 1{,}0$ is adduced in fig. 2. The figure shows that the head portion of the compression wave when passing through the interface $\xi = 0{,}5$ is partially reflected and propagated at $\tau > 0{,}5$ in the first layer as a tensile wave.

Conclusions

As expected, the domain of components interface of heterogeneous strip is most prone to damage,as close to the surface of the connection of structure components the thermo-elastic stress waves during reflection of diffraction experience the finite discontinuity and become tension waves.

The proposed method of calculation enables to conduct the computational experiments. By varying the geometric and mechanical parameters it is possible in a variety of loads to achieve smoothing of thermal

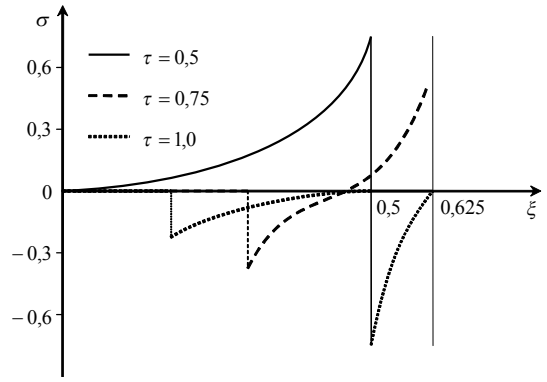

Fig. 2. Distribution of stresses along the strip thickness at different time instants

stress surges in the area of components connection of multilayer structures.

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Мастиновський Ю. В. Термопружні хвилі у двошаровій смузі, складеної з різнорідних матеріалів

Досліджується розповсюдження та дифракція термопружних хвиль у кусково-однорідній смузі, складеної із ізотропних матеріалів з різноманітними механічними властивостями. Одна з поверхонь конструкції зазнає дію термічного та механічного ударів. Припускається, що швидкість розповсюдження тепла скінченна. Наводяться рівняння зв'язаної термопружності та умови спряження в області контакту смуг. Задача розв'язується чисельно з використанням методу характеристик. Результати розрахунків для однорідної смуги добре узгоджуються з відомими, які були отримані іншими методами. Аналізуються явища відбиття та проходження термопружних хвиль на межі розділення шарів.

***Ключові слова**: термопружні хвилі, напруження, швидкість, метод характеристик.*

Мастиновский Ю.В. Термоупругие волны в двухслойной полосе, составленной из разнородных материалов

Исследуется распространение и дифракция термоупругих волн в кусочно-однородной полосе, составленной из изотропных материалов с различными механическими свойствами. Одна из поверхностей конструкции подвергается действию термического и механического ударов. Предполагается, что скорость распространения тепла конечна. Приводятся уравнения связанной термоупругости и условия сопряжения решений в области контакта слоев. Задача решается численно с применением метода характеристик. Результаты расчетов для однородной полосы хорошо согласуются с известными, полученными другими методами. Анализируются явления отражения и прохождения термоупругих волн на границе раздела слоев.

***Ключевые слова:** термоупругие волны, напряжения, скорость, метод характеристик.*